

Modeling Nonlinear Resonators Comprising Graphene: A Coupled Mode Theory Approach

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Abstract: We develop a perturbation theory framework for modeling nonlinear resonators comprising dispersive sheet materials. It is applied to model optical bistability with graphene-based nonlinear resonant structures in the THz and near-infrared regimes.

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1. Introduction and Framework

A general framework combining temporal coupled mode theory (CMT) with perturbation theory for examining nonlinear resonators comprising anisotropic and dispersive 3D (bulk) and 2D (sheet) materials (without the need for an equivalent bulk representation) is developed, expanding the existing literature focusing on bulk nonlinearities [1, 2]. Conductive sheet materials are modeled with the introduction of a nonlinear current term, next to the more common nonlinear polarization term. In the end, the nonlinear response can be obtained by solving a CMT first-order differential equation whose coefficients are specified through *linear* full-wave simulations. As a result, the framework is very efficient allowing for modeling large physical systems, performing elaborate design processes, and cross-checking with experimental results. In addition, it is very accurate as shown by comparing with full-wave nonlinear simulations.

Importantly, in the process of deriving the framework very interesting physics are revealed. Specifically, we find that media with complex conductivity disturb the equality of electric and magnetic energies on resonance; a condition which is typically taken for granted. This stems from the reactive power associated with the imaginary part of the (surface) conductivity. In addition, we demonstrate that the dispersive nature of conductive materials must be always taken into account, since it significantly impacts the nonlinear response. Interestingly, this is attributed to the energy that is stored in the (surface) current which is erroneously zeroed-out when dispersion is not taken into account.

Graphene, a prominent example of conductive sheet materials, has drawn considerable attention in recent years. It is already being employed in resonant structures to introduce nonlinearity and tuning. The proposed formulation is, thus, applied to model optical bistability with graphene-based 2D and 3D nonlinear resonators in the THz and near infrared (NIR) regimes. Graphene nonlinearity is of electronic nature, modeled with a $\bar{\sigma}_s^{(3)}$ surface conductivity tensor. High-quality bistable response is obtained with very low input powers, indicating the potential of graphene for nonlinear applications.

2. Graphene-based Resonant Structures for Bistability

We first focus on a simple 2D system consisting of an infinite graphene (or carbon) tube acting as a traveling-wave (TW) resonator and an infinite graphene sheet acting as the bus waveguide [Fig. 1(a)]. The combination of third-order nonlinearity and feedback leads to the manifestation of bistability for appropriate input power levels provided that the frequency detuning exceeds a specific threshold [2]. Then, there can be two possible output states depending on the history of the system. This is demonstrated in the bistable loop depicted in Fig. 1(c) for the system under study. Importantly, only 10 W/m of input power are required for accessing the bistable regime. For a three dimensional component with a $\lambda/2$ length along the direction of invariance, the input power is only 150 μ W at the frequency of 10 THz indicating the potential of graphene for nonlinear applications [3]. Note also that taking into account the dispersion of graphene's surface conductivity is crucial since it significantly affects the span and position of the bistable loop. Finally, the results obtained with the proposed framework are compared with full-wave nonlinear simulation [markers in Fig. 1(c)], indicating excellent agreement.

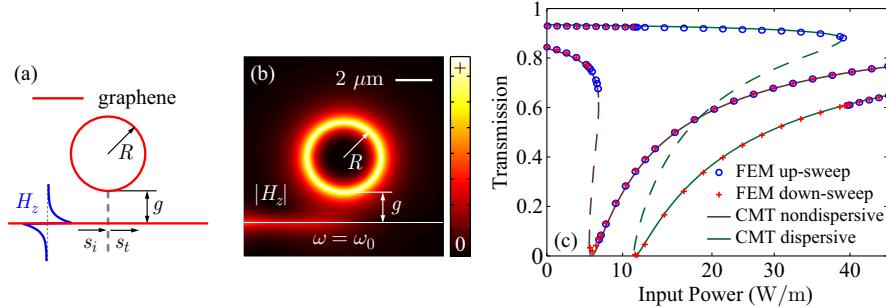


Fig. 1. (a) Graphene TW resonator, coupled with a graphene bus waveguide. (b) Magnetic field distribution (linear regime) on the 4th azimuthal order resonance ($R = 2 \mu\text{m}$, $g = 1.74 \mu\text{m}$, $f_0 \sim 10 \text{ THz}$) (c) Bistability curves obtained from both CMT and NL-VFEM indicating excellent agreement. Graphene dispersion significantly affects the nonlinear response and must be taken into account.

Having established the validity of the proposed framework, we can turn to practical 3D systems which would be too costly to model with full-wave nonlinear simulations. We first study a resonator-waveguide system based on a graphene nanoribbon (GNR), Fig. 2(a), in the THz regime. The waveguide and resonator structures are “written” on a uniform graphene sheet by properly patterning the Si substrate (not shown) leading to different chemical potential (and thus conductivity) values [4]. Optical bistability is obtained with extremely low input powers in the order of tens of μW . Next, we turn to the NIR regime and a resonant system based on a nonlinear Si-slot waveguide [5]. A uniform graphene sheet is placed on top, Fig. 2(b), to enhance the nonlinear response and allow for tunability. The contributions of sheet (graphene) and bulk (nonlinear polymer and silicon) materials to the overall nonlinearity are quantified. High-quality bistable response is attained with input powers in the order of few hundreds of mW.

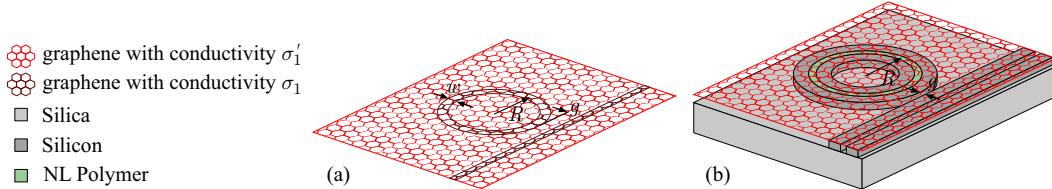


Fig. 2. (a) GNR ring resonator coupled with a GNR waveguide, “written” on a uniform graphene sheet of different conductivity. (b) Nonlinear Si-slot ring resonator coupled with a bus waveguide. A graphene sheet is placed on top to enhance the nonlinear response and allow for tunability.

3. Conclusions

In conclusion, we have developed a general mathematical framework for analyzing nonlinear resonant structures comprising anisotropic, dispersive bulk and sheet materials. We have applied it to model optical bistability with graphene-based resonant structures in the THz and NIR regimes, demonstrating low-power bistable response.

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References

1. M. Soljačić, M. Ibanescu, S. G. Johnson, Y. Fink, J. D. Joannopoulos, Phys. Rev. E **66**, 055601 (2002).
2. O. Tsilipakos, T. Christopoulos, E. E. Kriezis, J. Lightw. Technol. **34**, 1333 (2016).
3. T. Christopoulos, O. Tsilipakos, N. Grivas, E. E. Kriezis, Phys. Rev. E, paper accepted for publication
4. J. Zheng, L. Yu., S. He., D. Dai, Sci. Rep. **5**, 7987 (2015).
5. C. Koos *et al.*, Nat. Photon. **3**, 216-219 (2009).